The reliability of a two identical components cold standby system with replaceable repair equipment and multiple failure modes

Wenqing Wu [1*], Yaxing He [1], Xinyu Kuang [2], Shaojun Lan [2], Jinyin Liu [2]

[1] *Faculty of Science, Civil Aviation Flight University of China, Deyang 618307, China*;

[2] *School of Mathematics and Statistics, Sichuan University of Science and Engineering, Yibin 644002, China.*

A R T I C L E I N F O

Keywords:
Cold standby system; replace able repair facility; multiple f ailure modes; reliability

A B S T R A C T

This paper investigates a cold standby repairable system consisting of two identical components, where the working component is subject to c failure modes. The failed component is repaired by the repair equipment which may fail during the repair process. Using the Markov renewal process theory and the Laplace-Stieltjes transform, we derive the expression of the system reliability, and then numerical inversion of Laplace transform is used to discuss the time-dependent behavior of system reliability.

1.Introduction

In engineering systems such as aerospace, nuclear power, and flight control, redundancy design is commonly adopted to ensure system reliability. Cold standby is an important redundancy design, which describes a scenario where standby components remain completely inactive with no performance degradation or failure risk until activated. As technological systems grow increasingly complex, they face diverse failure modes caused by hardware defects, software errors, environmental factors, or human intervention. Consequently, system failures may manifest in a wide variety of forms. In fact, reliability models with multiple failure modes represent a natural extension of traditional binary-state reliability models, providing a more precise and refined characterization of system behavior. However, existing research predominantly assumes systems have only two (e.g., open-circuit/short-circuit) or three (e.g., hardware/software/human -induced) failure modes, with limited studies addressing reliability models incorporating more diverse failure mode classifications. Furthermore, in reliability system studies, when an operational component fails, repair technicians typically utilize specific tools (referred to as repair equipment) to restore the faulty component. During the repair process, the repair equipment itself may fail due to aging, wear and tear, or environmental factors. In such cases, the failed repair equipment must be replaced before the repair of the original faulty component can resume. This scenario represents a more general repairable system framework, and many classical reliability models can be viewed as special cases.

Wang et al. [1] investigated a warm standby repairable system consisting of two dissimilar units and a repairman, where unit 1 has priority. Under the assumption that the repair time of failed units follows a general distribution, they employed Markov renewal process theory, the Laplace transforms, and the Cramer ' s rule to analyze three key reliability measures, i.e., the mean time to the first failure, the system availability, and the expected number of failures within (0, t]. Furthermore, they examined the effect of system parameters on these steady-state reliability measures. Jiang and Tang [2, 3] introduced the strategies of "replaceable repair equipment" and "delayed repair" respectively into a two identical units parallel repairable system with two types of failure modes, where the repair time of failed units follows a general distribution. By employing the Markov renewal process theory and the Laplace transforms, they analyzed a series of steady-state reliability measures. Wei and Tang [4] investigated a cold standby repairable system with two dissimilar units with replaceable repair equipment and delayed repair. Utilizing Markov renewal process theory,they analyzed seve-

*Corresponding author

E-mailaddresses: wwqing0704@163.com(WenqingWu)

Received May 4,2025; Received in revised form May 8,2025; Accepted May 14,2025

ral reliability metrics including the steady-state system availability, the mean time to the first failure, the failure frequency, and the probability of waiting for repair. Additionally, numerical examples were provided to illustrate the corrections of theoretical results. Li et al. [5] investigated transient reliability measures for a cold standby system withtwo dissimilar units featuring priority, imperfect repair, and general repair time distribution. Their analysis focused on reliability function, and distribution function of the number of cycles within (0, t]. Wu et al. [6] examined a k-out-of-n(G) system with replaceable repair equipment, where the repair time of the failed units and the replacement time of the repair equipment obey a general distribution, and the repair equipment has c possible failure states. Using Markovrenewal process theory, the authors derived recursive expressions for the steady state system availability, the steady state failure frequency, the mean time to the first failure, the probability of repair equipment being in failed state, as well as its failure frequency.

Based on these studies, this paper investigates a cold standby repairable system consisting of two identical units with replaceable repair equipment and the operating component has c failure modes. By employing the Markov renewal process theory and the Laplace-Stieltjes transform, we derive the expressions of system reliability along with some special cases. Additionally, numerical examples are provided

2 Model descriptions and the semi-Markov Kernel

This section presents the model descriptions and its corresponding semi-Markov kernel for a two identical components cold standby system with replaceable equipment and multiple failure modes.

2.1 Model description

1) The system consists of two independent and identical components. At initial time point, both components are new of which one component works and the other in cold standby. When the working component fails, the standby component immediately transitions to working state.

2) The component lifetime X follows an exponential distribution $F(t)=P\{X\le t\}=1-e^{-\lambda t}$, $t\ge 0,\lambda\ge 0$. During operation, each component may fail according to one of c failure modes, where the probability of the ith failure mode occurring is p_i , and $\sum_{i=1}^{c}p_i=1$. For a component failing in the ith failure mode, its repair time Y_i follows a general distribution $G_i(t)=P\{Y_i\le t\},t\ge 0$ with mean repair time $E[Y_i]=\int_0^\infty t dG_i(t),\ t\ge 0, i=1,2,\cdots,c$.

3) The lifetime U of the repair equipment follows an exponential distribution with parameter $U(t)=P\{U\le t\}=1-e^{-\alpha t},t\ge 0,\alpha\ge 0$. During the repair of a failed component, the repair equipment itself may fail. Once fails, it is immediately replaced by a new one, and the replacement time V follows a general distribution $V(t)=P\{V\le t\},t\ge 0$.

4) Failed components are repaired on a first-failed-first-repaired basis. Each repaired component restored to an as-good-as-new condition. When the repair equipment fails, the repair resumes only after the repair equipment is replaced, and accumulated repair time remains valid.

5) The random variables $X,Y_i(i=1,2,\cdots,c),U,$ and V are mutually independent.

2.2 Semi-Markov Kernel of the System

Let $\tilde{Y}_i\,(i=1,2,\ldots,c)$ denote the generalized repair time of a failed component, which represents the total duration from the initiation of repair until its completion, including the time of replacing the repair equipment due to its failure. Let $\tilde{G}_i(t)=P\{\tilde{Y}_i\le t\},t\ge 0$,then

$$
\begin{aligned}
\tilde{G}_i(t) &= \sum_{l=0}^{\infty}P\{\tilde{Y}_i\le t, \text{the repairfequipment fails exactly } l \text{ times'during the repair process}\} \\
&= \sum_{l=0}^{\infty}P\left\{Y_i+\sum_{j=1}^{l}V_j\le t,\ \sum_{j=1}^{l}U_j\le Y_i<\sum_{j=1}^{l+1}U_j\right\} \\
&= \sum_{l=0}^{\infty}\int_0^t V^{(l)}(t-x)\frac{(\alpha x)^l}{l!}e^{-\alpha x}dG_i(x).
\end{aligned}
\tag{1}
$$

Applying the Laplace-Stieltjes transform to $\tilde{G}_i(t)$, we obtain

$$\tilde{g}_i(s) = \int_0^\infty e^{-st} d\tilde{G}_i(t) = g_i(s + \alpha - \alpha v(s)) \quad , \tag{2}$$

where $v(s) = \int_0^\infty e^{-st} dV(t)$.

Furthermore, the mean generalized repair time is

$$E[\tilde{Y}_i] = -\frac{d}{ds}\tilde{g}_i(s)\bigg|_{s=0} = E[Y_i](1+\alpha E[V]), i = 1, 2, \ldots, c \tag{3}$$

Based on the model descriptions, the system may occupy the following states:
0: There are no failed components, one component is operational, the other one is in cold standby;

$(1, i), i = 1, 2, \ldots, c$: One component is operational, the other failed one with type ?? failure mode;

$(2, i, j), i, j = 1, 2, \ldots, c$: Both components failed with type ?? and type j failure modes, respectively.

Since repair times follow general distributions, the system exhibits the Markov property only at some specific epochs. Thus, we adopt the Markov renewal process theory to analyze this model. Let $Z(t)$ denote the system state at time ?? and T_m be the time epoch of the ??-th state transition, where $T_0 = 0$ .Then define $Z_m = Z(T_m + 0)$ as the state entered immediately after the m-th transition. For $t \geq 0$, the semi-Markov kernel is given by

$$Q_{\varsigma,\kappa}(t) = P\{Z_{m+1} = \kappa, T_{m+1} - T_m \leq t \mid Z_m = \varsigma\}$$

where ς and κ are all possible system states.

We now derive the semi-Markov kernel expressions for each state transition scenario.

a). Transition from state 0 to state $(1, i), i = 1, 2, \ldots, c$

$$\begin{aligned} Q_{0,(1,i)}(t) &= P\{Z_{m+1} = (1,i), T_{m+1} - T_m \leq t \mid Z_m = 0\} \\ &= P\{X \leq t, the\ working\ component\ fails\ with\ type\ i\ mode\} \\ &= p_i(1 - e^{-\lambda t}), i = 1, 2, \cdots, c. \end{aligned} \tag{4}$$

b). Transition from state $(1, i), i = 1, 2, \ldots, c$ to state 0

$$\begin{aligned} Q_{(1,i),0}(t) &= P\{Z_{m+1} = 0, T_{m+1} - T_m \leq t \mid Z_m = (1,i)\} \\ &= P\{X > \tilde{Y}_i, \tilde{Y}_i \leq t\} \\ &= \int_0^t [1 - F(x)] d\tilde{G}_i(x), i = 1, 2, \cdots, c. \end{aligned} \tag{5}$$

c). Transition from state $(1, i)$ to state $(2, i, j), i, j = 1, 2, \cdots, c$

$$\begin{aligned} Q_{(1,i),(2,i,j)}(t) &= P\left\{Z_{m+1}=(2,i,j), T_{m+1}-T_m \le t \mid Z_m=(1,i)\right\} \\ &= P\left\{\tilde{Y}_i > X, X \le t, \text{and working component fails with type } j \text{ mode}\right\} \\ &= p_j \int_0^t \left[1-\tilde{G}_i(x)\right] dF(x), i, j = 1,2,\cdots,c. \end{aligned} \tag{6}$$

Applying the Laplace-Stieltjes transform to these semi-Markov kernels yield

$$q_{0,(1,i)}(s) = \frac{\lambda p_i}{s+\lambda}, i=1,2,\cdots,c \tag{7}$$

$$q_{(1,i),0}(s) = \tilde{g}_i(s+\lambda) = g_i\left(s+\lambda+\alpha-\alpha v(s)\right), i=1,2,\cdots,c \tag{8}$$

$$\begin{aligned} q_{(1,i),(2,i,j)}(s) &= \frac{\lambda p_j}{s+\lambda}\left[1-\tilde{g}_i(s+\lambda)\right] \\ &= \frac{\lambda p_j}{s+\lambda}\left[1-g_i(s+\lambda+\alpha-\alpha v(s))\right], i, j = 1,2,\cdots,c \end{aligned} \tag{9}$$

3. The system reliability

Let τ denote the time to the first system failure, then the distribution with different initial states is $\Psi_\zeta(t) = P\{\tau \le t | Z_0 = \zeta\}$, where $\zeta \in \{0,(0,i), i=1,2,\dots c\}$. Thus system Reliability is given by $R_\zeta(t) = P\{\tau > t | Z_0 = \zeta\} = 1-\Psi_\zeta(t)$

Theorem 1: The Laplace-Stieltjes transform of system reliability $R_\zeta(t)$ is $r_\zeta(s) = \int_0^\infty e^{-st} dR_\zeta(s)$, $\zeta \in \{0,(0,i), i=1,2,\dots c\}$ having the following expressions

$$r_0(s) = \frac{\dfrac{s^2+2\lambda}{(s+\lambda)^2} - \dfrac{s}{s+\lambda}\dfrac{\lambda}{s+\lambda}\sum_{i=1}^{c} p_i g_i\left(s+\lambda+\alpha-\alpha v(s)\right)}{1-\dfrac{\lambda}{s+\lambda}\sum_{i=1}^{c} p_i g_i\left(s+\lambda+\alpha-\alpha v(s)\right)} \tag{10}$$

$$r_{(1,i)}(s) = \frac{s}{s+\lambda} + \frac{\dfrac{s}{s+\lambda}\dfrac{\lambda}{s+\lambda} g_i\left(s+\lambda+\alpha-\alpha v(s)\right)}{1-\dfrac{\lambda}{s+\lambda}\sum_{i=1}^{c} p_i g_i\left(s+\lambda+\alpha-\alpha v(s)\right)}, i=1,2,\dots,c \tag{11}$$

Proof: To compute the system reliability, we first derive the expression of the distribution function $\Psi_\zeta(t) = P\{\tau \le t | Z_0 = \zeta\}$.

①. When $\varsigma = 0$, both components are new. One is operational, and the other is in cold standby, it gets

$$\begin{aligned}\Psi_0(t) &= P\{\tau \le t, T_1 > t \mid Z_0 = 0\} + P\{\tau \le t, T_1 \le t \mid Z_0 = 0\} \\ &= \sum_{i=1}^{c} \int_0^t P\{\tau \le t - x \mid Z_1 = (1,i)\} dP\{Z_1 = (1,i), T_1 \le x \mid Z_0 = 0\} \\ &= \sum_{i=1}^{c} \int_0^t \Psi_{(1,i)}(t-x) dQ_{0,(1,i)}(x).\end{aligned} \tag{12}$$

②. When $\varsigma = (1,i), i = 1, 2, \ldots, c$, one component is operational, the other is failed with Type-?? failure mode, then

$$\begin{aligned}\Psi_{(1,i)}(t) &= P\{\tau \le t, T_1 > t \mid Z_0 = (1,i)\} + P\{\tau \le t, T_1 \le t \mid Z_0 = (1,i)\} \\ &= \int_0^t P\{\tau \le t - x \mid Z_1 = 0\} dP\{Z_1 = 0, T_1 \le x \mid Z_0 = (1,i)\} \\ &\quad + \sum_{i=1}^{c} P\{\tau = X, X \le t, \tilde{Y}_i > X, \textit{and working component fails with type } j \textit{ failure}\} \\ &= \int_0^t \Psi_0(t-x) dQ_{(1,i),0}(x) + \sum_{j=1}^{c} Q_{(1,i),(2,i,j)}(t).\end{aligned} \tag{13}$$

Applying the Laplace-Stieltjes transform to the distribution functions $\Psi_0(t)$ and $\Psi_{(1,i)}(t), i = 1, 2, \ldots, c$ yields

$$\psi_0(s) = \sum_{i=1}^{c} \psi_{(1,i)}(s) q_{0,(1,i)}(s) = \frac{\lambda}{s+\lambda} \sum_{i=1}^{c} p_i \psi_{(1,i)}(s) \tag{14}$$

$$\begin{aligned}\psi_{(1,i)}(s) &= \psi_0(s) q_{(1,i),0}(s) + \sum_{j=1}^{c} q_{(1,i),(2,i,j)}(s) \\ &= \psi_0(s) g_i(s + \lambda + \alpha - \alpha v(s)) + \frac{\lambda}{s+\lambda}\left[1 - g_i(s + \lambda + \alpha - \alpha v(s))\right], i = 1, 2, \ldots, c.\end{aligned} \tag{15}$$

$$\psi_0(s) = \frac{\left(\dfrac{\lambda}{s+\lambda}\right)^2 \left[1 - \sum_{i=1}^{c} p_i g_i(s+\lambda+\alpha-\alpha v(s))\right]}{1 - \dfrac{\lambda}{s+\lambda} \sum_{i=1}^{c} p_i g_i(s+\lambda+\alpha-\alpha v(s))} \tag{16}$$

$$\psi_{(1,i)}(s) = \frac{\lambda}{s+\lambda} - \frac{\dfrac{s}{s+\lambda} \dfrac{\lambda}{s+\lambda} g_i(s+\lambda+\alpha-\alpha v(s))}{1 - \dfrac{\lambda}{s+\lambda} \sum_{i=1}^{c} p_i g_i(s+\lambda+\alpha-\alpha v(s))}, i = 1, 2, \ldots, c \tag{17}$$

From the relation $r_\varsigma(s) = 1 - \psi_\varsigma(s)$, we obtain the Laplace-Stieltjes transform of the system reliability.

Corollary 1: When $c = 2$, the model degenerates to a cold standby repairable system with two identical components, one replaceable repair facility, and two types of failure modes. The Laplace-Stieltjes transform of the reliability functions are

$$r_0(s)=\frac{\frac{s^2+2\lambda}{(s+\lambda)^2}-\frac{s\lambda}{(s+\lambda)^2}\left[p_1g_1(s+\lambda+\alpha-\alpha v(s))+p_2g_2(s+\lambda+\alpha-\alpha v(s))\right]}{1-\frac{\lambda}{s+\lambda}\left[p_1g_1(s+\lambda+\alpha-\alpha v(s))+p_2g_2(s+\lambda+\alpha-\alpha v(s))\right]}$$

$$r_{(1,1)}(s)=\frac{s}{s+\lambda}+\frac{\frac{s}{s+\lambda}\frac{\lambda}{s+\lambda}g_1(s+\lambda+\alpha-\alpha v(s))}{1-\frac{\lambda}{s+\lambda}\left[p_1g_1(s+\lambda+\alpha-\alpha v(s))+p_2g_2(s+\lambda+\alpha-\alpha v(s))\right]}$$

$$r_{(1,2)}(s)=\frac{s}{s+\lambda}+\frac{\frac{s}{s+\lambda}\frac{\lambda}{s+\lambda}g_2(s+\lambda+\alpha-\alpha v(s))}{1-\frac{\lambda}{s+\lambda}\left[p_1g_1(s+\lambda+\alpha-\alpha v(s))+p_2g_2(s+\lambda+\alpha-\alpha v(s))\right]}$$

4 Numerical examples

From Theorem 1, we can know it is almost impossible to provide expressions for system transient reliability. Therefore, this section analyzes the transient system reliability using the relationship between the Laplace transform and the Laplace-Stieltjes transforms, combined with the numerical inverse of the Laplace transform technique.

First, assuming that the replacement time for repairing equipment obeys the deterministic distribution, namely,

$$V(t)=P\{V\le t\}=\begin{cases}0, & t\le 1/\beta,\\ 1, & t>1/\beta,\end{cases} \quad \text{, then} \quad v(s)=e^{-s/\beta} \quad , \quad E[V]=\frac{1}{\beta} \quad .$$

Consider working component having 4 types of failure modes, the distribution of the repair time for each failure mode is

(1) Exponential distribution

$$G_1(t)=P\{Y_1\le t\}=1-e^{-\mu_1 t}, t\ge 0, \mu_1\ge 0 \quad \text{, then} \quad \tilde{g}_1(s)=\frac{\mu_1}{s+\lambda+\alpha-\alpha e^{-s/\beta}+\mu_1} \quad .$$

(2) Deterministic distribution

$$G_2(t)=P\{Y_2\le t\}=\begin{cases}0, & t\le 1/\mu_2,\\ 1, & t>1/\mu_2,\end{cases} \quad \text{then} \quad \tilde{g}_2(s)=e^{-\left(s+\lambda+\alpha-\alpha e^{-s/\beta}\right)/\mu_2} \quad .$$

(3) Gamma distribution

$$G_3(t)=P\{Y_3\le t\}=\frac{\gamma(\sigma_3,\ \mu_3 t)}{\Gamma(\sigma_3)}, \mu_3\ge 0, \sigma_3>0$$, where $\Gamma(\sigma_3)=\int_0^\infty t^{\sigma_3-1}e^{-t}dt$ is gamma function,

$\gamma(\sigma_3,\mu_3 t)=\int_0^{\mu_3 t}u^{\sigma_3-1}e^{-u}du$ is incomplete gamma function. From the properties of the gamma distribution function,

it gets $g_3(s)=\left(\frac{\mu_3}{s+\lambda+\alpha-\alpha e^{-s/\beta}+\mu_3}\right)^{\sigma_3}$.

(4) Phase-Type distribution

$$G_4(t) = P\{Y_4, \ t\} = 1 - \sigma_4 e^{T_4 t} e_{4\times1} \quad ,$$

where $\sigma_4 = (0.25, 0.5, 0.15, 0.15)$, $\mathbf{T}_4 = \begin{pmatrix} -15 & 2 & 1 & 11 \\ 1 & -6 & 2 & 1 \\ 2 & 1 & -7 & 2 \\ 1 & 1.5 & 0 & -5.5 \end{pmatrix}$, $\mathbf{T}_4^0 = \begin{pmatrix} 1 \\ 2 \\ 2 \\ 3 \end{pmatrix}$, $e_{4\times1} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$,

and $\tilde{g}_4(s) = \sigma_4 \left(\left(s + \lambda + \alpha - \alpha e^{-s/\beta} \right) \mathbf{I}_{4\times4} - \mathbf{T}_4 \right)^{-1} \mathbf{T}_4^0$

Further, we select parameters $\lambda = 0.65, p_1 = \frac{11}{23}, \ p_2 = \frac{7}{23}, \ p_3 = \frac{4}{23}, \ p_4 = \frac{1}{23}$, $\mu_1 = 0.25, \mu_2 = 0.80$, $\sigma_3 = 2/3, \mu_3 = 0.75, \alpha = 0.2, \beta = 0.5$. With these expression and parameters, a numerical computation program was executed, and the corresponding results are shown in Table 1 and Figure 1 below.

Table 1. The values of system transient reliability.

	t=0.5	t=1.5	t=3	t=5	t=10
R0(t)	0.9601	0.7715	0.4944	0.2704	0.0796
R(1,1)(t)	0.7356	0.4283	0.2127	0.1091	0.0375
R(1,2) (t)	0.7225	0.4104	0.2374	0.1706	0.0645
R(1,3) (t)	0.7932	0.5657	0.3539	0.2103	0.0679
R(1,4) (t)	0.7636	0.6307	0.4196	0.2523	0.0777

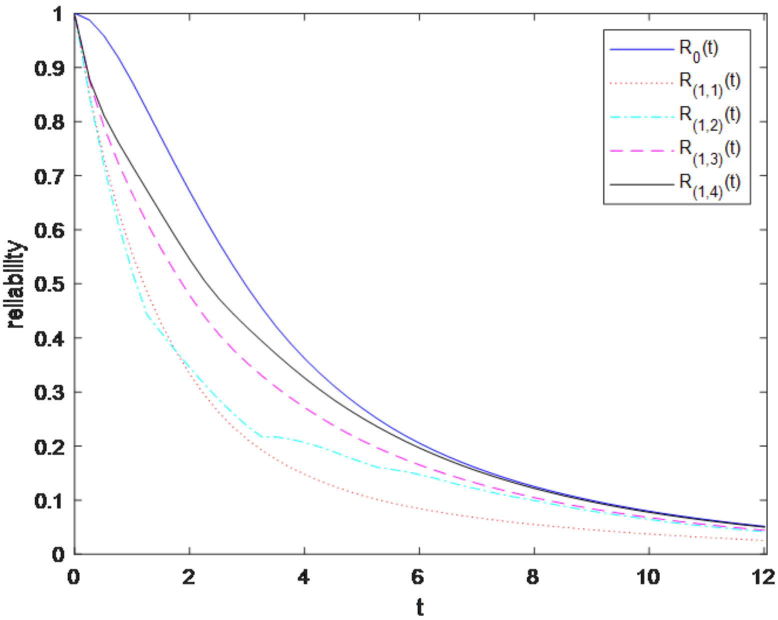

Figure 1: Illustration of system reliability under different initial conditions

From the calculation results, it can be seen that the transient reliability of the system gradually decreases over time and tends towards 0, which is consistent with the actual situation. At time 0, the system reliability is 1, which is determined by the model assumption that both components in the system are new, with one in working condition and the other in cold standby.

With these parameters, we can directly calculate the mean generalized repair time which are $E\left[\tilde{Y}_1\right]=1.75, E\left[\tilde{Y}_2\right]=1.87$, $E\left[\tilde{Y}_3\right]=1.24, E\left[\tilde{Y}_4\right]=0.40$. And the results show that $E\left[\tilde{Y}_1\right]>E\left[\tilde{Y}_2\right]>E\left[\tilde{Y}_3\right]>E\left[\tilde{Y}_4\right]$ indicating that the repair time varies for different types of failure modes. From Table 1 and Figure 1 that $R_0(t)>R_{(1,4)}(t)>R_{(1,3)}(t)>R_{(1,2)}(t)>R_{(1,1)}(t)$. The shorter the generalized repair time, the higher the system reliability.

5. Conclusion

This article studies two identical components cold standby system with multiple types of failure modes and one replaceable repair equipment, where the repair time under different failure modes follows a general distribution. The system reliability are discussed using the Markov renewal process theory and the Laplace-Stieltjes transform.

In the numerical example, assuming that the repair time of the component follows an exponential distribution, deterministic distribution, gamma distribution, and phase type distribution. By using the numerical inverse of the Laplace transform, the numerical results of the system transient reliability is given. In the future, we will consider the system transient reliability measures for more complex system with multiple failure modes.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

All authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This work is supported by the National Natural Science Foundation of China (No.72001181).

Reference

[1] Wang JT, Xie N, Yang N, Reliability analysis of a two-dissimilar-unit warm standby repairable system with priority in use, Communications in Statistics—Theory and Methods, 2021, 50(4): 792-814.
[2] Jiang Y，Tang YH，Analysis of parallel repairable system with a replaceable repair facility and two types of failure units，Mathematica Applicata，2014，27(3)：535-545.(In Chinese)
[3] Jiang Y，Tang YH，Two types of failure units paralleled repairable system with a replaceable facility and delay repair，Mathematics in Practice and Theory，2014，44(7)：174-186.(In Chinese)
[4] Wei YY，Tang YH，Two units cold standby repairable system with a replaceable repair facility and delay repair，Chinese Journal of Engineering Mathematics，2020，37(4): 423-441.(In Chinese)
[5] Li Guanchen, Kagaris Dimitri, Time-domain computation of the reliability of standby systems with and without priority under general repair, Computers & Industrial Engineering, 2024, 189, 109983.
[6] Wu WQ, Tang YH, Lan SJ, Analysis of k/n(G) repairable system with replaceable repair facility, Acta Mathematica Sinica, Chinese Series, 2016, 59(6): 799-820.(In Chinese)
[7] Cao JH, Cheng K, Introduction to Reliability Mathematical, Higher Education Press, Beijing, 2006. (In Chinese)